- 19. Prove that u and v be two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at a interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c. Then the function f = u + iv has a derivative at c. Moreover $f'(c) = D_1 u(c) + iD v(c)$.
- 20. A quadric surface with center at the origin has the equation $Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1, \text{ Find the lengths of its semi-axes.}$

NOVEMBER/DECEMBER 2023

GMA22 — REAL ANALYSIS-II

Time: Three hours

Maximum: 75 marks

SECTION A — $(10 \times 2 = 20 \text{ marks})$

Answer ALL questions.

- Define Lebesgue Integral.
- 2. Define Monotonic Sequences.
- 3. Define Riemann-integrable.
- 4. Define Measurable Functions.
- Define Orthogonal Systems.
- 6. Define Dirichlet Integrals.
- 7. Directional Derivative.
- 8. Write down Taylor's formula.
- 9. Prove that u + iv is a complex-valued function with a derivative at a point z in $Cin f(z) = |f'(z)|^2$.
- 10. State Inverse Function theorem.

SECTION B — $(5 \times 5 = 25 \text{ marks})$

Answer ALL questions.

11. (a) Prove that $f \in U(I)$ and $g \in U(I)$, and if f(x) = g(x) almost everywhere on I, Then $\int f = \int g$.

Or

- (b) Prove that $f \in U(I)$ and $\{s_n\}$ and $\{t_m\}$ be two sequences generating f. Then $\lim_{n \to \infty} \int s_n = \lim_{n \to \infty} \int t_m$.
- 12. (a) Prove that $f \in M(I)$ and if $|f(x)| \le g(x)$ almost everywhere on I for some nonnegative g in L(I), then $f \in L(I)$.

Or

- (b) Prove that $f \in L(I)$ and if f is bounded almost everywhere on I, then $f^2 \in L(I)$.
- 13. (a) Prove that g is of bounded variation on $[0, \delta]$,

 Then $\lim_{\alpha \to +\infty} \frac{2}{\pi} \int_{0}^{\delta} g(t) \frac{\sin \alpha t}{t} dt$ is equal to g(0+).

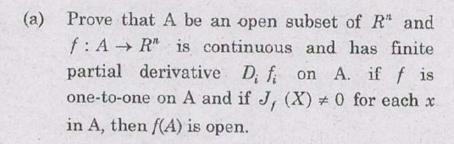
Or

(b) Show that $x = \pi - 2 \sum_{n=1}^{\infty} \frac{\sin nx}{n}$ if $0 < x < 2\pi$.

14. (a) Prove that S be an open connected subset of R^n , and let $f: S \to R^m$ be differentiable at each point at each point of S. If f'(c) = 0 for each c in S, Then f is constant on S.

Or

(b) State and Prove that Taylor's theorem.



Or

(b) State and prove Implicit function theorem.SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

- 16. State and Prove Lebesgue dominated convergence theorem.
- 17. State and Prove Riesz-Fischer theorem.
- 18. State and Prove Weierstrass approximation theorem.

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